Comparison Between Nonparametric and Parametric Estimate of the Conditional Intensity Function of a Seismic Space-Time Point Process

Confronto tra la stima parametrica e non parametrica della funzione di intensità condizionata di un processo sismico di punto spazio-temporale

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1. Introduction

A seismic gap can be defined as a segment of an active geologic fault that has not produced seismic events for an unusually long time; gaps are often considered susceptible to future strong earthquakes occurrence and therefore their identification may be useful for predictive purposes. In this paper we try to identify gaps in an area of South Tyrrhenian Sea by comparing the observed seismicity, estimated by nonparametric method, and the theoretical one, described by a particular space-time point process (ETAS model).

2. Nonparametric and parametric estimation

A number of statistical models have been proposed for representing the intensity function of earthquakes. The simpler models assume that earthquakes occur in space and time according to a stationary point process, such that conditional rate becomes a constant. In seismology, however, the stationarity hypothesis might be acceptable only with respect to time, because epicenters usually display a substantial degree of spatial heterogeneity and clustering. Description of seismic events often requires the definition of more complex models than stationary Poisson process and the relaxation of the assumption of statistical independence of earthquakes. Therefore, second-order properties of point processes may have a relevant role in the study and the comprehension of seismic process and its realization. Indeed, when aggregation is present, it is useful to introduce some generalizations of the simple Poisson process, such as self-exciting point processes, to model events that are clustered together, and self-correcting processes when regularities are observed, e.g. the strain-release model (Daley and Vere-Jones, 2003). A widely used model is ETAS model (Ogata and Tanemura, 2003), that is a self-exciting point process, describing earthquakes activity, in a given region during a period of time, through a branching structure.

Also in this field, the parametric models estimation suffers by many drawbacks, often related to the definition of a reliable mathematical model from the geophysical theory and
to the sensitivity of statistical estimates to the composition of the sample, that is the space-time region under study. Many of the disadvantages of the parametric modelling can be avoided by making use of nonparametric techniques, such as kernel density methods (Silverman, 1986). Therefore a flexible model, that is useful in presence of several data for which a not immediately obvious discrimination between principal and secondary events is not reliable, estimated by nonparametric method is proposed; in particular a three dimensional Gaussian kernel estimator is used. Then, by a graphical approach, we compare the ETAS model, estimated by ML with the kernel estimate of the seismic activity of the South Tyrhrenian Sea from 1981 to 2005 (fig. 1 on the left, for latitudetime space), by considering their ratio (fig. 1 on the right). In the space-time image below, regions with ratio values smaller than one are identified by a brighter grey, while regions with a ratio larger than 1 are identified by a darker grey: darker areas indicate that the observed seismicity is smaller than those calculated by the estimated space-time ETAS model. From this plot, some darker grey area around the source region before large earthquakes that induced a big sequence of events may be observed. Although this result may suggest the existence of seismic gaps before large earthquakes, we consider this method just as a starting analysis to deal with this kind of issue, since it needs further data to be available and the application of more rigorous methodology (e.g. tests to assess the significance of variation of the space-time seismic activity). On the other hand, even if further studies are required to get a quantitative interpretation of such kind of gaps and to analyze their relationship with the main events of a sequence, we think that it may provide a useful starting-point for studies aimed to seismic hazard evaluation.

**Figure 1: Nonparametric intensity estimate of the seismic activity of the South Tyrhrenian Sea from 1981 to 2005 (on the left) and ratio between the parametric (ETAS) and nonparametric ones (on the right)**

References

