Modelling Skewness of Multivariate Financial Time Series

Modellizzazione dell’asimmetria di serie storiche finanziarie multivariate

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1. Introduction

According to some authors, skewness in financial markets is caused by feedback effects (French et al., 1987) which make falls in stock prices due to bad news greater than rises of the same stocks due to good news. De Luca and Loperfido (2004) and De Luca et al. (2005) formalized the concept of feedback effect in univariate and multivariate settings by introducing the univariate and multivariate SGARCH models, respectively. In the latter model the feedback effect is modelled through a vector of nonnegative parameters (referred to as the feedback parameter). De Luca et al. (2005) focused on multivariate skewness as measured by Mardia’s index, and not on the third cumulants, as it is done in the present work.

2. Third cumulant

There is some empirical evidence that small capitalized financial markets are negatively skewed, in the sense that their third centered moments (i.e. their third cumulants) are negative. In the multivariate case, the notion of third cumulants can be generalized as follows, when all necessary moments are finite:

\[ M_3^d = E[(y - \mu) \otimes (y - \mu)^T \otimes (y - \mu)] \in \mathbb{R}^{d^3} \times \mathbb{R}^d \]  

(1)

where \( y \) is a \( d \)-dimensional random vector of with expectation \( \mu \) and \( \otimes \) denotes the Kronecker (tensor) product. Its statistical analogue is

\[ M_3^n = \frac{1}{n} \sum_{i=1}^{n} (x_i - m) \otimes (x_i - m)^T \otimes (x_i - m) \in \mathbb{R}^{d^3} \times \mathbb{R}^d \]  

(2)

where \( x_i \) is the transpose of the \( i \)-th row of the \( n \times d \) data matrix \( X \) and \( m \) is the mean vector. When the data matrix \( X \) contains the daily log-returns of small financial markets, the corresponding third sample cumulants is often a matrix with negative entries. For

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example, the sample third cumulants of daily log-returns corresponding to the indices DAX, IBEX and MIB from 01/01/2001 30/11/2007 is

\[
\begin{pmatrix}
-0.3944 & -0.2014 & -0.3961 \\
-0.2014 & -0.0920 & -0.2415 \\
-0.3961 & -0.2415 & -0.4141 \\
-0.2014 & -0.0920 & -0.2415 \\
-0.0920 & -0.0881 & -0.1455 \\
-0.2415 & -0.1455 & -0.2160 \\
-0.3961 & -0.2415 & -0.4141 \\
-0.2145 & -0.1455 & -0.2160 \\
-0.4141 & -0.2160 & -0.4458
\end{pmatrix}
\times 10^{-6}
\] (3)

The corresponding standard deviations are much greater, but inferential implications are not straightforward, since it is well known that the $t$-test can be very conservative when the parent distribution is long-tailed, as it is the case of financial data.

3. Theoretical results

Empirical results in the last section are consistent with those in De Luca and Loperfido (2007), as well as with the following theoretical property of the multivariate SGARCH model:

**Proposition:** The third cumulant of a random vector belonging to a multivariate SGARCH process is a matrix whose entries are never positive. Moreover, if all components of the feedback parameter are positive (null), the third cumulants is a negative (null) matrix.

References


