Environmental Monitoring and Measurement Errors:  
Some Statistical Developments

Monitoraggio ambientale e errori di misura: alcuni sviluppi

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Riassunto: Algoritmi di monitoraggio come le carte di controllo Shewhart o le carte CUSUM sono spesso usate nel monitoraggio ambientale. E’ noto che le proprietà statistiche di questi strumenti sono influenzate in modo sensibile dalla presenza di errori di misura. Studi recenti hanno sottolineato che nelle rilevazioni ambientali è opportuno considerare la presenza di un particolare errore di misura denominato "a due componenti". Nel presente lavoro si studia tramite simulazioni l'effetto dell'errore di misura a due componenti sulle prestazioni della carta di controllo Shewhart per il livello medio. I risultati ottenuti mostrano che tale errore influenza in modo sensibile sulle prestazioni della carta di controllo.

Keywords: measurement errors, environmental monitoring, Shewhart control charts.

1. Introduction

Monitoring algorithms, such as the Shewhart and Cusum control charts or the GLR algorithm, are often used for surveillance purposes in the field of environmental studies (Gibbons, 1999, Bordignon and Scagliarini, 2000). Recent studies (Rocke et al., 2003) have shown the important role played by the two components measurement error model in environmental analysis. Monitoring algorithms can be very responsive to measurement errors, and studies in statistical quality control offer a series of methods of taking into account errors in decision rule design (Mittag and Stemann, 1998, Linna and Woodal, 2001). Similar studies on the two-components error model, which is particularly suited to environmental applications, are not available however. The aim of this paper is to try to help fill this gap by studying the effects of the two components error model on the performance of the Shewhart control chart for the mean level. Results of simulation studies reveal that gauge imprecision may seriously increase the false alarm rate.

2. Measurement errors in environmental studies

It is widely recognized that when measuring pollutants, it is appropriated to take into account two types of measurement errors: the first is a constant error, while the second is a proportional one. Moreover, many measurement technologies require a linear

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calibration curve in order to estimate the actual concentration of an analyte in a sample (Rocke and Lorenzato, 1995). This leads to the following model

\[ Y^*_\mu = \alpha + \beta \mu e^\eta + \varepsilon \]  

(1)

where: \( Y^*_\mu \) is the response at concentration \( \mu \); \( \alpha \) and \( \beta \) are the parameters of the calibration curve; \( \eta \sim \mathcal{N}(0, \sigma_\eta^2) \) represents the proportional error that is always present but is only noticeable at concentrations significantly above zero; and \( \varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2) \) represents the additive error that is always present but is only really noticeable for near zero concentrations.

In (1) \( \mu \) is non random. This assumption may be acceptable for a laboratory sample, but in a real-world monitoring situation, one needs to assume that the concentration of a given pollutant, or a suitable transformation, constitutes a random variable \( X \sim \mathcal{N}({\mu}, {\sigma}^2) \).

Thus, we have

\[ Y^* = \alpha + \beta X e^\eta + \varepsilon \]  

(2)

Therefore the observable response \( Y^* \) is a random variable consisting of the sum of a normal variable, \( \varepsilon \), with the product of a normal variable, \( X \), and a log-normal variable \( e^\eta \). The mean and standard deviation of \( Y^* \) are, respectively:

\[ E(Y^*) = \alpha + \beta \mu \sqrt{e^{\sigma_\eta^2}} \]

and

\[ \sigma_{Y^*} = \left( \beta^2 \left( e^{\sigma_\eta^2} \sigma^2 + \mu^2 \left( e^{\sigma_\eta^2} - 1 \right) \right) + \sigma^2 \left( e^{\sigma_\eta^2} - 1 \right) \right)^{1/2} + \sigma_\varepsilon^2 \],

while the distribution of \( Y^* \) cannot be treated analytically. In the error-free situation, the response \( Y = \alpha + \beta X \) is normally distributed with mean \( E(Y) = \alpha + \beta \mu \) and standard deviation \( \sigma_Y = \beta \sigma \).

3. Control charts and two components error model

In the free error-free situation, the mean chart with 3-sigma limits used to monitor response \( Y \) is:

\[ CL = \alpha + \beta \mu ; \quad UCL = CL + 3 \frac{\sigma_Y}{\sqrt{n}} ; \quad LCL = CL - 3 \frac{\sigma_Y}{\sqrt{n}} \].

A detailed study of the effects of the additive error \( \varepsilon \) on the above control chart is contained in Linna and Woodall (2001). In the two-components error case, assuming all involved parameters to be known, the chart designed using the usual 3-sigma rule is:

\[ CL^* = CL^* - 3 \frac{\sigma_{Y^*}}{\sqrt{n}} ; \quad UCL^* = CL^* + 3 \frac{\sigma_{Y^*}}{\sqrt{n}} \].

The presence of the proportional error, \( \eta \), also influences the position of the central line. This situation is quite different from what was described by Linna and Woodall (2001), and since the distribution of response \( Y^* \) cannot be treated analytically, we have to study the performance of the control charts by means of simulation. We first examine the effects of measurement errors on the false positive rate. This is an important question, since false alarms in environmental uses of control charts could lead to a series of expensive and unnecessary actions or regulations. We set the following values in all of the simulations: \( n=5, \alpha=2, \beta=5, \mu=10 \) and \( \sigma=3 \) and each condition was replicated 10000 times. In the first experiment, the false positive rates
was studied as a function of the ratio $\sigma_{\mu}/\sigma_\epsilon$ for $\sigma_{\mu}/\sigma_\epsilon=0.02$ and 0.1. The results are summarised in Table 1.

**Table 1: False positive rates (L under LCL, U over UCL; $\sigma_{\mu}/\sigma_\epsilon=0.02$, $\sigma_{\mu}/\sigma_\epsilon=0.1$)**

<table>
<thead>
<tr>
<th>$\sigma_{\mu}/\sigma_\epsilon$</th>
<th>0.2</th>
<th>0.04</th>
<th>0.06</th>
<th>0.08</th>
<th>0.1</th>
<th>0.12</th>
<th>0.14</th>
<th>0.16</th>
<th>0.18</th>
<th>0.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_\mu$ L</td>
<td>0.0011</td>
<td>0.0013</td>
<td>0.0014</td>
<td>0.0012</td>
<td>0.0012</td>
<td>0.0011</td>
<td>0.0010</td>
<td>0.0015</td>
<td>0.0015</td>
<td>0.0012</td>
</tr>
<tr>
<td>$\sigma_\mu$ U</td>
<td>0.0016</td>
<td>0.0017</td>
<td>0.0017</td>
<td>0.0009</td>
<td>0.0017</td>
<td>0.0013</td>
<td>0.0013</td>
<td>0.0017</td>
<td>0.0008</td>
<td>0.0012</td>
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As a second check, we studied the effects of the measurement errors in the out-of-control condition. The Average Run Length (ARL) of a control chart denotes the mean delay in detecting a "shift" in the process. In environmental applications, the ARL evaluation is important since an excessively lengthy delay in detecting a problem could lead to dangerous consequences. In the second experiment, for several values of the ratio $\sigma_{\mu}/\sigma_\epsilon$ and for $\sigma_{\mu}/\sigma_\epsilon=0.02$ and $\sigma_{\mu}/\sigma_\epsilon=0.1$, out-of-control situations were simulated for known change instants $t_0$, by means of 10000 replications of the experiment for size shifts of $\pm 0.5\sigma_\mu$ and $\pm 1\sigma_\mu$. The aim of the simulation was to evaluate the mean detection delay time, which is an approximation of the ARL function. The results are shown in Table 2.

**Table 2: Mean detection delay time (ARL) ($\sigma_{\mu}/\sigma_\epsilon=0.02$ and $\sigma_{\mu}/\sigma_\epsilon=0.1$)**

<table>
<thead>
<tr>
<th>$\sigma_{\mu}/\sigma_\epsilon$</th>
<th>0.2</th>
<th>0.04</th>
<th>0.06</th>
<th>0.08</th>
<th>0.1</th>
<th>0.12</th>
<th>0.14</th>
<th>0.16</th>
<th>0.18</th>
<th>0.2</th>
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<tbody>
<tr>
<td>$\sigma_\mu$ L</td>
<td>34</td>
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<td>34</td>
<td>34</td>
<td>34</td>
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<td>35</td>
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<tr>
<td>$\sigma_\mu$ U</td>
<td>34</td>
<td>36</td>
<td>38</td>
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<td>43</td>
<td>48</td>
<td>56</td>
<td>65</td>
<td>78</td>
<td>94</td>
</tr>
</tbody>
</table>

Since $\sigma_\epsilon$ and $\sigma_\eta$ indicate the precision of a measuring device, the above-mentioned experiments enabled us to study the effects of different precision structures on control-chart performance. To complete our study, we considered the case of a pseudo-
population representing a real situation by developing the toluene example reported in Rocke and Lorenzato (1995) where $\alpha=11.51$, $\beta=1.524$, $\sigma_\epsilon=5.698$, $\sigma_\eta=0.1032$, and concentration ranges from 5 picograms to 15 nanograms in 100 $\mu$L. The "under control" and "out of control" situations were simulated for different values of the coefficient of variation (c.v.($X$)) of concentration $X$. In this way, for a given precision structure, we reproduce a set of different working situations for the measurement device. The results for $\mu=5$ and $\mu=15000$ are summarised in Table 3.

4. Analysis and conclusions

We have outlined the important role played by the proportional error component. While the additive error basically inflates the variance of the observable response $Y^e$, the proportional error also modifies the expected value of $Y^e$, thus $Y^e$ becomes a non-symmetric random variable. This is confirmed by the chart performances: when $\sigma_\eta$ is relatively large with respect to $\sigma_\epsilon$ ($\sigma_\eta/\sigma_\epsilon=0.1$) the number of false alarms over UCL is greater than the number under LCL. Results also indicate that the false-alarm rate is highly responsive to the value of standard deviation $\sigma_\eta$ (Table 1), and that the false alarm rates in the presence of measurement errors are higher than they are in the error-free situation. Moreover, the effects of $\sigma_\eta$ on the false-alarm rate are more significant for high concentration values (Table 3). We ought to point out that for the out-of-control situation in the error-free case, for shifts of $\pm 0.5$ standard deviation units the ARL value for the 3-sigma mean chart is 33, while for shifts of $\pm 1$ standard deviation units, the ARL is approximately 4. Simulation results show that for small negative shifts, the presence of measurement errors penalizes the performance of the chart. Once again, the mean detection delay time is responsive to the value of standard deviation $\sigma_\eta$ (Table 2) and to the degree of concentration (Table 3), in particular for small values of the coefficient of variation. Our analysis shows, as was expected, that the two-component error model has a substantial effect on the statistical properties of a control chart. This fact underlines the importance of taking errors into account when setting up the chart if erroneous conclusions are to be avoided.

References